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Improved lumped models for combined convective and radiative cooling of a wall

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ABSTRACT

Improved lumped parameter models are developed for the transient heat conduction of a wall subjected to combined convective and radiative cooling. The improved lumped models are obtained through two point Hermite approximations for integrals. It is shown by comparison with numerical solution of the original distributed parameter model that the higher order lumped model ($H_{1,1}/H_{0,0}$ approximation) yields significant improvement of average temperature prediction over the classical lumped model.

1. Introduction

Transient heat conduction in a solid body with combined convective and radiative cooling or heating on the surface has been studied due to its relevance in various technological applications such as dynamical thermal behaviour of walls, aerodynamic heating of spaceships and satellites, nuclear reactor thermohydraulics and glass manufacture [1–6].

In one of the earliest work on the subject, Haji-Sheikh and Sparrow [7] used the Monte Carlo technique to obtain solutions for a plate subjected to simultaneous boundary convection and radiation. Crosbie and Viskanta [8] analysed transient cooling and heating of a plate by combined convection and radiation. The transient heat conduction equation and the boundary conditions are transformed to a non-linear Volterra integral equation of the second kind for the surface temperature. Davies [9] applied the heat balance integral technique to obtain an approximate solution for the general conditions of plate in a non-zero temperature environment. Sundén [10] presented numerical solutions based on finite difference method of the thermal response of a composite slab subjected to a time-varying incident heat flux on one side and combined convective and radiative cooling on the other side. Later, Sundén [11] applied the same technique to assess the thermal response of a circular cylindrical shell due to a time-varying incident surface heat flux while cooled by combined convection and radiation. Parang et al. [5] solved the problem of inward solidification of a liquid in cylindrical and spherical geometries due to combined convective and radiative cooling by the regular perturbation method. Recently, Liao et al. [12] obtained an explicit series solution of a non-linear model of combined convective and radiative cooling of a spherical body with temperature-dependent thermal conductivity using the homotopy analysis approach.

The lumped parameter approach has been widely used in the analysis of the dynamical thermal behaviour of buildings [13–16,3,17]. As in the analysis of other complex thermal systems, this classical approach is extremely useful and sometimes even mandatory when a simplified formulation of the transient heat conduction is sought. As an inherent limitation of the lumped parameter approach, moderate to low temperature gradient within the region is assumed, which, through the associated problem parameters, governs the accuracy of such approximate formulations. As a rule of thumb, the classical lumped parameter approach, where uniform temperature is assumed within the region, is in general restricted to problems with Biot number less than 0.1. In most building energy simulation problems, the Biot number is much higher [3]. In other words, the moderate to low temperature gradient assumption is not reasonable in such applications, thus more accurate approach should be adopted. To overcome the limitations of the classical lumped model, improved lumped models have been developed by different approaches [18–23]. Cotta and Mikhailov [18] proposed a systematic formalism to provide improved lumped parameter formulation for steady and transient heat conduction problems based on Hermite approximations for integrals that define averaged temperatures and heat fluxes. This approach has been shown to be efficient in a great variety of practical applications [24–26].

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Nomenclature

c_p	specific heat
h	convective heat transfer coefficient
k	thermal conductivity
L	thickness of the wall
N_{rc}	the radiation–conduction parameter
T	temperature
T_a	adiabatic temperature
T_f	right-side fluid temperature
T_i	initial temperature
T_m	left-side fluid temperature
T_s	radiation sink temperature
t	time

Greek symbols

α	thermal diffusivity
η	dimensionless spatial coordinate
ϵ	surface emissivity
ρ	density
σ	Stefan–Boltzmann constant
τ	dimensionless time
θ	dimensionless temperature

Subscripts

1	left-side surface
2	right-side surface
av	average

In this work, we present improved lumped models for transient combined convective and radiative cooling of a wall, extending previous works on the particular cases of asymmetric convective cooling [19] and radiative cooling [20]. The proposed lumped models are obtained through two point Hermite approximations for integrals [27,18]. By comparing with numerical solution of the original distributed parameter formulation, it is shown that the higher order improved lumped model ($H_{1,1}/H_{0,0}$ approximation) yields significant improvement of average temperature prediction over the classical lumped model.

2. The mathematical formulation

Let us consider the dynamical thermal behaviour of a wall subjected to convective heat transfer at one side and combined convective and radiative heat transfer at the other side. The wall is modeled as a one dimensional slab of finite thickness L, initially at a uniform temperature T_i. It is assumed that the thermophysical properties of the wall are homogeneous, isotropic and independent of the temperature. At $t = 0$, the wall is exposed to an environment of a constant fluid temperature T_m with a constant convective heat transfer coefficient h_1 at the left-side, and an environment of a constant fluid temperature T_f with a constant heat transfer coefficient h_2 and a constant radiation sink temperature T_s at the right-side.

The mathematical formulation of the problem is given by

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}, \quad \text{in } 0 < x < L, \text{ for } t > 0, \tag{1}$$

with initial and boundary conditions taken as

$$T(x,0) = T_i, \quad \text{in } 0 < x < L, \quad \text{at} \quad t = 0, \tag{2}$$

$$-k\frac{\partial T}{\partial x} = h_1(T_m - T), \quad \text{at} \quad x = 0, \quad \text{for } t > 0, \tag{3}$$

$$-k\frac{\partial T}{\partial x} = h_2(T - T_f) + \epsilon\sigma(T^4 - T_s^4), \quad \text{at} \quad x = L, \quad \text{for } t > 0, \tag{4}$$

where T is the temperature, t the time, x the spatial coordinate, $\alpha(= k/\rho c_p)$ the thermal diffusivity of the wall, k the thermal conductivity, ϵ the surface emissivity, and σ the Stefan–Boltzmann constant.

It should be noted that in general the environmental fluid temperature T_f differs from the radiation sink temperature. It is convenient to introduce the adiabatic surface temperature T_a, defined by

$$h_2(T_a - T_f) + \epsilon\sigma(T_a^4 - T_s^4) = 0 \tag{5}$$

The boundary condition Eq. (4) can be rewritten with use of the adiabatic surface temperature

$$-k\frac{\partial T}{\partial x} = h_2(T - T_a) + \epsilon\sigma(T^4 - T_a^4), \quad \text{at} \quad x = L, \quad \text{for } t > 0, \tag{6}$$

The mathematical formulation (1)–(6) can now be rewritten in dimensionless form as follows:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2}, \quad \text{in} \quad 0 < \eta < 1, \quad \text{for } \tau > 0, \tag{7}$$

$$\theta(\eta, 0) = 1, \quad \text{in} \quad 0 < \eta < 1, \text{ at } \tau = 0, \tag{8}$$

$$-\frac{\partial \theta}{\partial \eta} = Bi_1(\theta_m - \theta), \quad \text{at} \quad \eta = 0, \quad \text{for } \tau > 0, \tag{9}$$

$$-\frac{\partial \theta}{\partial \eta} = Bi_2(\theta - \theta_a) + N_{rc}(\theta^4 - \theta_a^4), \quad \text{at} \quad \eta = 1, \quad \text{for } \tau > 0, \tag{10}$$

where the dimensionless parameters are defined by

$$\theta = \frac{T}{T_i}, \quad \eta = \frac{x}{L}, \quad \tau = \frac{\alpha t}{L^2}$$

$$Bi_1 = \frac{h_1 L}{k}, \quad Bi_2 = \frac{h_2 L}{k}, \quad N_{rc} = \frac{\epsilon\sigma L T_i^3}{k}.$$

It can be seen that the problem is governed by five dimensionless parameters, θ_m, θ_a, Bi_1, Bi_2 and N_{rc}. The radiation–conduction parameter, N_{rc} that governs the radiative cooling, is conceptually analog to the Biot number, Bi, which is the governing parameter for an equivalent transient convective cooling.

3. Lumped models

Let us introduce the spatially averaged dimensionless temperature as follows:

$$\theta_{av}(\tau) = \int_0^1 \theta(\eta, \tau) d\eta. \tag{11}$$

Operating Eq. (7) by $\int_0^1 d\eta$ and using the definition of average temperature, Eq. (11), we get

$$\frac{d\theta_{av}(\tau)}{d\tau} = \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=1} - \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}. \tag{12}$$

Now, when the boundary conditions Eqs. (9) and (10) are used, we have

$$\frac{d\theta_{av}(\tau)}{d\tau} = -Bi_1[\theta(0,\tau) - \theta_m] - Bi_2[\theta(1,\tau) - \theta_a] - N_{rc}[\theta(1,\tau)^4 - \theta_a^4]. \tag{13}$$

Eq. (13) is an equivalent integro-differential formulation of the mathematical model, Eq. (7), with no approximation involved.

Supposing that the temperature gradient is sufficiently smooth over the whole spatial solution domain, the classical lumped system analysis (CLSA) is based on the assumption that the boundary temperatures can be reasonably well approximated by the average temperature, as

$\theta(0,\tau) \cong \theta(1,\tau) \cong \theta_{av}(\tau),$

which leads to the classical lumped model,

$$\frac{d\theta_{av}(\tau)}{d\tau} = -Bi_1(\theta_{av}(\tau) - \theta_m) - Bi_2(\theta_{av}(\tau) - \theta_a) - N_{rc}(\theta_{av}(\tau)^4 - \theta_a^4), \tag{14}$$

to be solved with the initial condition for the average temperature,

$$\theta_{av}(0) = 1. \tag{15}$$

In an attempt to enhance the approximation approach of the classical lumped model, we develop improved lumped models by providing better relations between the boundary temperature and the average temperature, based on Hermite-type approximations for integrals that define the average temperature and the heat flux. The general Hermite approximation for an integral, based on the values of the integrand and its derivatives at the integration limits, is written in the following form [27]:

$$\int_a^b y(x)dx = \sum_{v=0}^{\alpha} C_v y^{(v)}(a) + \sum_{v=0}^{\beta} D_v y^{(v)}(b),$$

where $y(x)$ and its derivatives $y^{(v)}(x)$ are defined for all $x \in (a,b)$. It is assumed that the numerical values of $y^{(v)}(a)$ for $v = 0,1,\ldots,\alpha$, and $y^{(v)}(b)$ for $v = 0,1,\ldots,\beta$ are available. The general expression for the $H_{\alpha,\beta}$ approximation is given by

$$\int_a^b y(x)dx = \sum_{nu=0}^{\alpha} C_v(\alpha,\beta)h^{\alpha+1}y^{(v)}(a) + \sum_{nu=0}^{\beta} C_v(\beta,\alpha)h^{\alpha+1}y^{(v)}(b) + O(h^{\alpha+\beta+3}),$$

where $h = b - a$, and

$$C_v(\alpha,\beta) = \frac{(\alpha+1)!(\alpha+\beta+1-v)!}{(v+1)!(\alpha-v)!(\alpha+\beta+2)!}.$$

We first employ the plain trapezoidal rule in the integrals for both average temperature and average heat flux ($H_{0,0}/H_{0,0}$ approximation), in the form

$$\theta_{av}(\tau) \cong \frac{1}{2}[\theta(0,\tau) + \theta(1,\tau)], \tag{16}$$

$$\int_0^1 \frac{\partial\theta(\eta,\tau)}{\partial\eta}d\eta = \theta(1,\tau) - \theta(0,\tau) \cong \frac{1}{2}\left[\frac{\partial\theta}{\partial\eta}|_{\eta=0} + \frac{\partial\theta}{\partial\eta}|_{\eta=1}\right]. \tag{17}$$

The boundary conditions (9) and (10) are substituted into Eq. (17) to yield

$$\theta(1,\tau) - \theta(0,\tau) = \frac{1}{2}[-Bi_1(\theta_m - \theta(0,\tau)) - Bi_2(\theta(1,\tau) - \theta_a) + N_{rc}(\theta(1,\tau)^4 - \theta_a^4)]. \tag{18}$$

The boundary temperature $\theta(0,\tau)$ is solved from Eq. (16) and substituted into Eq. (13)

$$\frac{d\theta_{av}(\tau)}{d\tau} = -Bi_1(2\theta_{av} - \theta(1,\tau) - \theta_m) - Bi_2(\theta(1,\tau) - \theta_a) - N_{rc}(\theta(1,\tau)^4 - \theta_a^4), \tag{19}$$

The boundary temperature $\theta(0,\tau)$ is solved from Eq. (16) and substituted into Eq. (18) and we obtain an equation that relates $\theta(1,\tau)$ to $\theta_{av}(\tau)$

$$N_{rc}\theta(1,\tau)^4 + (4 + Bi_1 + Bi_2)\theta(1,\tau) - (4 + 2Bi_1)\theta_{av}(\tau) - N_{rc}\theta_a^4 + Bi_1\theta_m - Bi_2\theta_a = 0. \tag{20}$$

Analytical solution of Eq. (20) is readily obtained by using a symbolic computation software such as *Mathematica* and then used to close the ordinary differential equation (19) for the average temperature, to be solved with the initial condition, Eq. (15), providing the $H_{0,0}/H_{0,0}$ model.

Then we further improve the lumped model by employing two-side corrected trapezoidal rule in the integral for average temperature, in the form

$$\theta_{av}(\tau) \cong \frac{1}{2}[\theta(0,\tau) + \theta(1,\tau)] + \frac{1}{12}\left[\frac{\partial\theta}{\partial\eta}|_{\eta=0} - \frac{\partial\theta}{\partial\eta}|_{\eta=1}\right], \tag{21}$$

The boundary conditions (9) and (10) are substituted into Eq. (21) to yield

$$\theta_{av}(\tau) \cong \frac{1}{2}[\theta(0,\tau) + \theta(1,\tau)] + \frac{1}{12}[-Bi_1(\theta_m - \theta(0,\tau)) + Bi_2(\theta(1,\tau) - \theta_a) + N_{rc}(\theta(1,\tau)^4 - \theta_a^4)], \tag{22}$$

while keeping the plain trapezoidal rule in the integral for heat flux ($H_{1,1}/H_{0,0}$ approximation).

The boundary temperature $\theta(0,\tau)$ is solved from Eq. (22)

$$\theta(0,\tau) = \frac{12\theta_{av}(\tau) - (6 + Bi_2)\theta(1,\tau) - N_{rc}\theta(1,\tau)^4 + N_{rc}\theta_a^4 + Bi_1\theta_m + Bi_2\theta_a}{6 + Bi_1}, \tag{23}$$

Substituted Eq. (23) into Eq. (18), we obtain an equation that relates $\theta(1,\tau)$ to $\theta_{av}(\tau)$

$$(4 + Bi_1)N_{rc}\theta(1,\tau)^4 + (12 + 4(Bi_1 + Bi_2) + Bi_1Bi_2)\theta(1,\tau) - (12 + 2Bi_1)\theta_{av}(\tau) - (4 + Bi_1)N_{rc}\theta_a^4 - (4 + Bi_1)Bi_2\theta_a + 2Bi_1\theta_m = 0. \tag{24}$$

Similarly, Eq. (23) and the analytical solution of Eq. (24) is obtained and used to close the ordinary differential Eq. (13) for the average temperature, to be solved with the initial condition Eq. (15), providing the $H_{1,1}/H_{0,0}$ model.

4. Numerical results and discussions

The solutions of classical and improved lumped models are shown in graphical form in comparison with a reference finite difference solution of the original distributed model, Eqs. (7)–(10). The initial boundary value problem defined by Eqs. (7)–(10) is solved by using an implicit finite difference method, with a 201 nodes mesh in the radial direction and a dimensionless time step of 0.00001 for all cases. Different values of the Biot numbers Bi_1 and Bi_2, and the radiation–conduction parameter N_{rc} are chosen so as to assess the range of application of the lumped models.

Two particular cases of the problem can be identified. If $\theta_m = \theta_a = 0$, $Bi_1 = N_{rc} = 0$, the problem reduces to the classical problem of a slab subjected to symmetric convective cooling with Bi_2 as the only dimensionless parameter. This case has been discussed by Cotta and Mikhailov [18]. If $Bi_1 \neq Bi_2$, $\theta_m = \theta_a = 0$, $N_{rc} = 0$, the case of asymmetric convective cooling was discussed by Su [19]. In both cases, Eqs. (20) and (24) reduce to linear relations between $\theta(1,\tau)$ and $\theta_{av}(\tau)$. A particular case of radiative cooling of a spherical body was considered by Su [20]. As both symmetric and asymmetric convective cooling has been discussed in previous work, we consider here only cases in which $N_{rc} \neq 0$ and $Bi_2 \neq 0$, when one side of the slab is subjected to combined convective and radiative cooling. Without loss of generality, we take $\theta_m = 0.8$ and $\theta_a = 0.5$ in the following examples. Fig. 1 shows that for $Bi_1 = Bi_2 = N_{rc} = 0.1$, all three lumped models, the classical, the $H_{0,0}/H_{0,0}$ and $H_{1,1}/H_{0,0}$, predict the time evolution of the average dimensionless temperature reasonably well, when compared with the prediction of the distributed parameter model. However, as shown in Figs. 2–7, the higher order lumped model ($H_{1,1}/H_{0,0}$ approximation) presents good agreement with the reference finite difference solution for values of Biot numbers as high as 20.0 and N_{rc} as high as 10.0, and the classical lumped model already deviates from the reference solution at $Bi_1 = Bi_2 = 1.0$ and $N_{rc} = 1.0$.

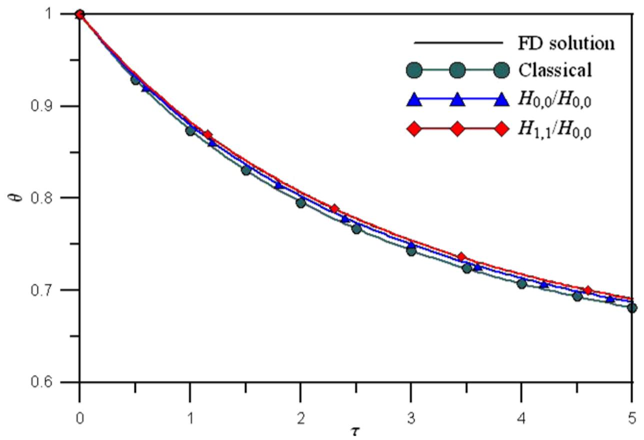

Fig. 1. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 0.1$, $Bi_2 = 0.1$ and $N_{rc} = 0.1$.

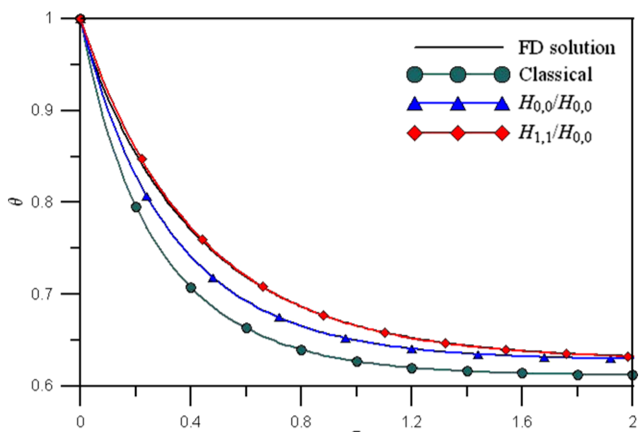

Fig. 2. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 1.0$, $Bi_2 = 1.0$ and $N_{rc} = 1.0$.

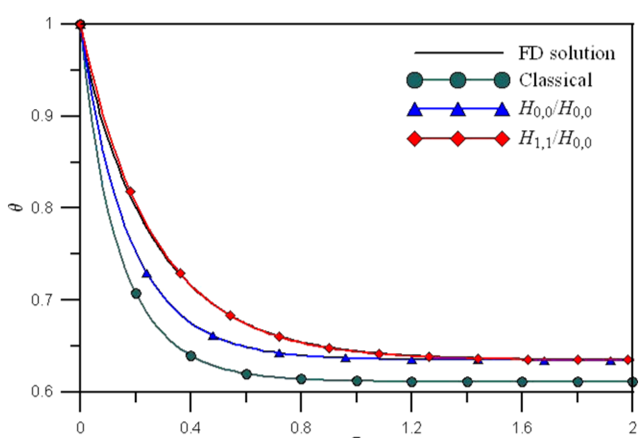

Fig. 3. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 2.0$, $Bi_2 = 2.0$ and $N_{rc} = 2.0$.

It is important to observe that although the lower order improved model ($H_{1,1}/H_{0,0}$) does not predict the average temperature accurately for higher values of Biot number and the radiation–conduction parameter, it predicts the correct value of the steady-state temperature. On the other hand, the classical lumped model gives systematically a lower value of the steady-state average temperature of the slab.

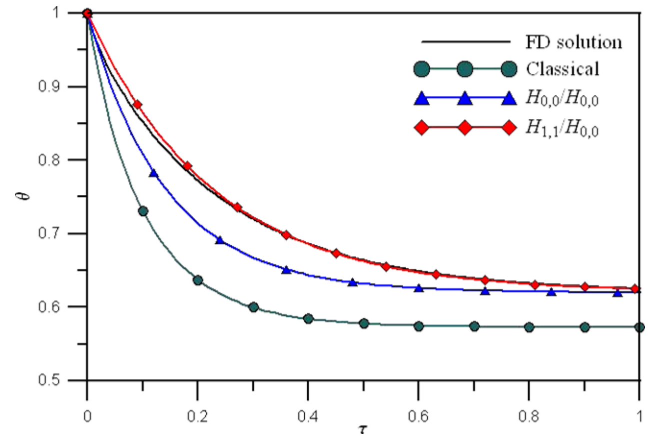

Fig. 4. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 2.0$, $Bi_2 = 5.0$ and $N_{rc} = 2.0$.

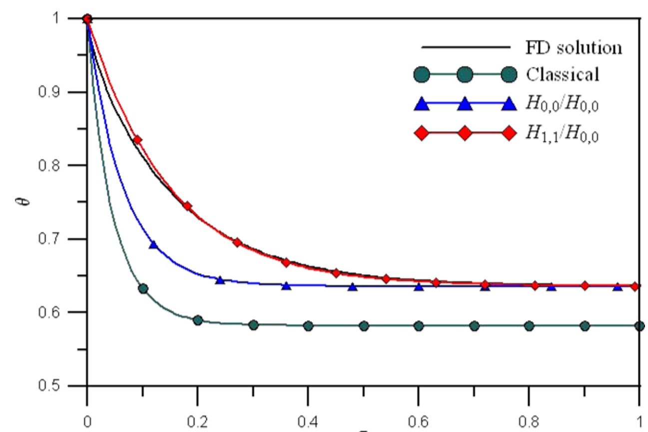

Fig. 5. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 5.0$, $Bi_2 = 10.0$ and $N_{rc} = 5.0$.

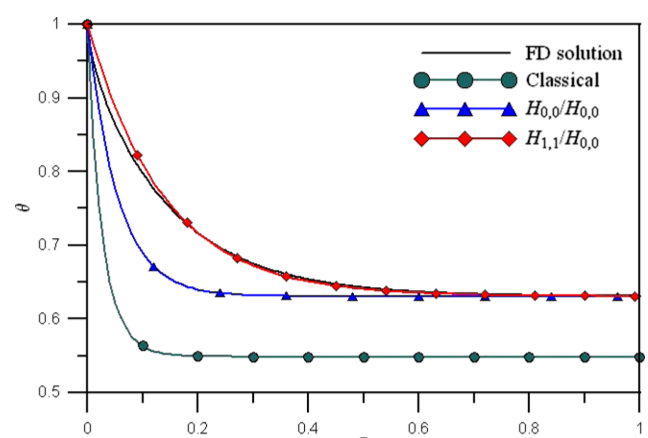

Fig. 6. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 5.0$, $Bi_2 = 20.0$ and $N_{rc} = 5.0$.

5. Conclusions

Improved lumped parameter models are developed for the transient heat conduction of a wall subjected to combined convective and radiative cooling. The improved lumped models are obtained through two point Hermite approximations for integrals. It is

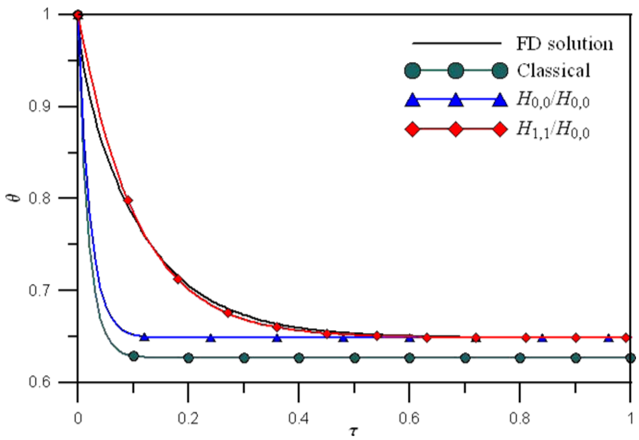

Fig. 7. Dimensionless temperature as a function of dimensionless time for $Bi_1 = 20.0$, $Bi_2 = 20.0$ and $N_{rc} = 10.0$.

shown by comparison with numerical solution of the original distributed parameter model that the higher order lumped model ($H_{1,1}/H_{0,0}$ approximation) yields significant improvement of average temperature prediction over the classical lumped model. It is concluded that the improved lumped model ($H_{1,1}/H_{0,0}$ approximation) can be used for the simulation of the dynamical thermal behaviour of a wall subjected to combined convective and radiative cooling for Biot numbers as high as 20.0 and the radiation–conduction parameter N_{rc} as high as 10.0.

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